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**ON MATHEMATICS IN ENERGY RESEARCH**

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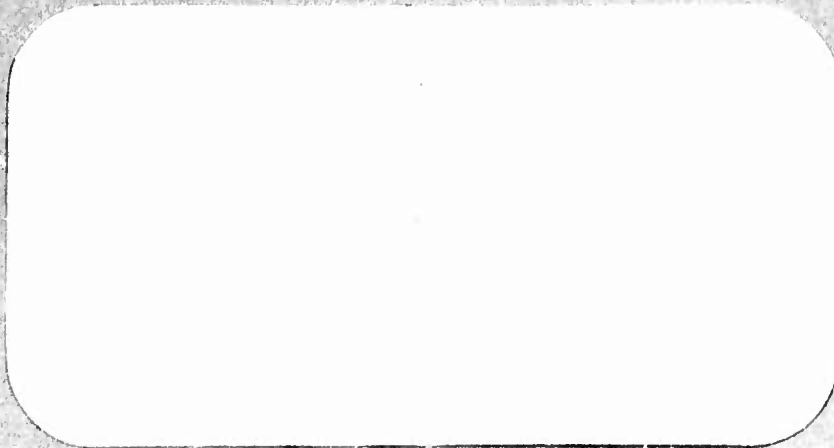
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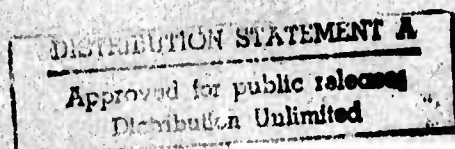
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by

William F. Lucas

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  This report contains comments on the papers presented at the SIMS Research Applications Conference on Energy at Alta, Utah on July 7-11, 1975, plus remarks on some types of mathematical models and new methodologies which may prove useful in such research.		

## ON MATHEMATICS IN ENERGY RESEARCH

W. F. Lucas\*

Introduction. I would like to comment in a general way on the role which mathematics may play in energy research, and with some attention given to the talks presented at this meeting. Although my own interest in energy research is quite recent, I did do some homework in advance by reading extensively in the area and by consulting with some Cornell colleagues who are working over a fairly broad range of different sorts of energy problems. When I suggested to the latter that mathematics might have a useful role in energy research, I was somewhat taken back by their replies to the effect that they rather doubted whether it would have any major effect. They did grant the need for better forecasting techniques, improved algorithms, and suchlike. Their responses surprised me since I had previously presumed that mathematics could make worthwhile contributions, and I am not at all convinced that the economists, lawyers, and others in Washington were doing such a great job of decisionmaking in this problem area. Although I still believe that there is an important place for a substantial mathematical component, I have become somewhat skeptical as to whether it will in fact advance to a point of major importance.

It would be quite accurate, and also more friendly, if I were to merely state that we have heard some useful and excellent research described here, that the mathematical approach is off to a good start and moving in the right directions regarding energy, and (of course) that there is need for additional research and funding in this area. However, I will attempt instead to play

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more the role of a devil's advocate and search for some faults, shortcomings and omissions. In so doing, I realize that it is much easier to be destructive and find fault than to criticize constructively, and that any fool can pose a great number of questions for which only a very brilliant person may be able to find a few answers. No doubt my remarks will also reflect some of my personal research interests.

There are many very obvious and desirable properties which research and good mathematical models should possess. A few of these will be mentioned here briefly, since much current research is in an early stage and some of it could benefit by containing some of these considerations. Theoretical scientists should frequently get into the "field" where the scene of action takes place; they should consult with and involve the applied experts at the grass-roots level at several stages of their model building; and they should learn something about the other specialties involved as is expected for example in the SIMS transplant program. It is important to do one's homework in the sense of searching to be sure that the required analytical techniques have not already been developed to analyze some other problem or in some alternate field. For example, distributions used in chemical or biological warfare should prove useful in studying exhaust pollution. When appropriate, models must be extended to include the necessary nonlinear, dynamic, and stochastic elements. Although it is normally impossible to include all future possibilities such as technological breakthroughs or war, it is most important to include improved forecasting techniques and analysis. There is also more need for additional sensitivity analysis and model validation for the models presented here. These above remarks are most obvious ones and are known to all model-builders. So I will proceed to some of my general reactions to the papers presented here,



and then on to discuss some mathematical directions which I believe could assist in this research.

General reactions. I am struck most by the fact that much of the current attention to and the funding of energy research is due to the fact that we have just been through an energy crisis, and a serious energy problem is still upon us. Yet most of the work presented here is what I would label as "crisis independent." Most of these investigations seem to presume rather standard operating conditions, and would have been useful activities to pursue even if there had been no crisis. It is always good practice to use the routine analytical tools to assist in the conserving or efficient use of energy. It appears that the United States has enough oil reserves to last a couple of decades, sufficient coal resources for a couple of centuries, and with some fortuitous advances ample nuclear energy for the more distant future. So there is time to work out a reasonable long-range energy plan. On the other hand, there have been many energy crises in the past few centuries, caused by the depletion or cutting off of previous sources as happened when wood for burning ran out in Europe or when the Suez Canal was closed in 1956. There will very likely be additional crises before long. So it appears desirable to have additional research directed toward the more immediate world situation, including questions on immediate policies and possible scenarios, as well as other aspects of a more political, sociological or behavioristic nature.

The upcoming decisions with respect to our energy policies will likely have great impact on our styles and quality of life. Nevertheless, I feel that most of the research discussed here will not provide much crucial input into the important short-range decisions to be made. Most of the models seem valid for at most a fairly short time period. Yet, as is usually the case, it will

take several years for the results to reach the level of serious consideration by the major decisionmakers, or some other world dynamics may alter the basic assumptions or validity of the models beforehand. I sense that the researchers here have a lack of real faith in their own models, since there seems to be very little proselytizing by the speakers for immediate implementation of their approaches or complaining about the failure of those people in Washington for not adopting their results. The remarks made at this meeting by Alta's Mayor Leavitt about the need for closing the huge gap between theoretical studies and implementation of decisions are most pertinent in this regard.

From a scientific point of view I am somewhat disappointed to see a shortage of entirely new mathematical methodology, or really ingenious uses of known mathematics. We have perhaps seen a little too much of the "well known tool looking for a new application." Investigators must confront the new and different aspects of our problems, break away from the old techniques when appropriate, and introduce the fresh and original models which are more likely over time to provide the really significant new insights. It seems unlikely that the routine methods of operations research, statistics, or classical applied mathematics will suffice to solve those problems with a significant social or behavioristic component in them.

It is easy to list several important aspects of the global energy picture which were not discussed at this conference nor are receiving enough mathematical attention. This list would include many environmental factors, greater consideration of new technologies, questions of peace and international relations, more strategic and policy matters, sociological impacts on various groups, etc. I personally would like to see additional projects investigating the full world oil market, and which would eventually be extended to include



money, food, technological assistance, and other exchangeable items. One very oversimplified view of the recent energy crisis is that a certain new and rather large coalition formed, and then merely pointed out that there is really a very large range over which the gains to the participants can be split. The "pie" to be divided is "larger," not because the Pareto surface has moved "up," but because the new threat point of noncooperation is further "down" than had previously been presumed. Oil markets are under study by Morris Adelman and others at M.I.T. and a fine survey of work in this direction was undertaken by Dietrich Fischer, Dermot Gately and J. F. Kyle at N.Y.U. At Cornell we have just begun to consider such markets using the market games introduced by L. S. Shapley and Martin Shubik in [3] and subsequent papers. By investigating various solution concepts from the theory of multiperson cooperative games, we hope to gain insights into coalitional values, cohesion of various alliances, and strength of basic negotiation positions.

More specific suggestions. Most of the studies on energy seem to me to exhibit a discomforting incompleteness or selectivity in the sense of omitting or not relating to major aspects of the problem. Rarely does one see an attempt to list all of the participants and potential groupings, all of the detailed choices, options and counteractions, nor all of the expected or conceivable outcomes or consequences, whether predictable or merely possible ones. This is an area in which it is crucial to avoid overspecialization and isolation, and instead to consider the many possible interrelations and side effects, those secondary and tertiary effects further down the road. More complete detailing of all of the variables, their interactive relationships, and the more global structural relations are called for. More complete and recursive contingency plans should be spelled out. For example, I am unaware of any

extensive study which really anticipates or fully traces through the many indirect effects that may follow from the rationing of gasoline in the United States. Unless isolated analytical studies are integrated within a larger framework, they will likely prove to be less valuable to the decisionmakers than those broader but less deep reports of a more expository or journalistic nature. For example, chapter 14 in the recent popular book [2] by John McDonald gives a most valuable discussion of the various economic and environmental effects likely to occur as a result of bringing refineries and supertankers to the State of Maine. More broadly inclusive studies, even at the sacrifice of mathematical depth and sophistication, may prove more beneficial at this early stage of energy research.

There have been some excellent models which have made good use of linear programming or other optimization techniques. It is clearly insufficient however to optimize only one particular objective function, as is often the case when one is minimizing some usage or cost factor or maximizing the gross national consumption under various constraints. Certainly, a multiplicity of objectives must be handled simultaneously. It is hardly sufficient to merely weigh each of the different objective functions in some ad hoc manner, and then to proceed to optimize the resulting function. Especially, as in many instances, when the objectives are "pulling" in more or less opposite directions from one another; for example, the typical choice between economical energy sources versus maintaining good environment. In general, a large domain of feasible or "efficient" solutions must be brought into any final consideration. Some recent work on social welfare functions and on various bargaining point solution concepts in the multiperson game theory may assist in selecting the desirable solution vector when one is confronted with programming problems with multiple objective

functions. Recent reports on this latter game theory topic have been written by Robert Rosenthal and Ehud Kalai (visiting) at Northwestern University, as well as by others.

A good deal of the work by mathematical economists on theoretical research on energy is concerned with equilibrium concepts. These results are quite informative and are often nonintuitive. For example, they determine the "optimal" price point or curve at which the OPEC cartel should price its oil, which in most models turns out to be well below the going rates. Such equilibrium solutions are self-reinforcing against unilateral deviations by one of the participants, and there are mathematical theorems which guarantee their existence in many realistic situations. From the pure mathematician's point of view, these are some of the most elegant models in current use. On the other hand, equilibrium results can in general exhibit many bad properties: non-uniqueness, existence only in mixed strategies which are difficult to interpret or employ, non Pareto optimality, and "prisoner dilemma" type outcomes. They may also not be stable with respect to small perturbations and algorithms converging to them may cycle. Such models sometimes assume perfectly competitive markets, and they may predict zero prices for commodities in oversupply or suggest the depleting of one source just at the time when a new or cheaper source comes into being. It is true that in many current models the equilibrium solutions do not have such faults or difficulties. And I would furthermore disagree with some oil experts who state that such models are irrelevant to the real world. Nevertheless, one should be aware of the many potential pitfalls when working with equilibrium models.

In contrast to the security, safety, stability and reinforcing ideas which one associates with equilibrium points, I would like to see additional research

into the more cooperative concepts such as fairness, sharing, equity and justice. Such results can often be arrived at by various bargaining, bidding, auctions and fair division schemes. Once negotiated, these new outcomes can become the "equilibria" in a new situation in which the "rules of the game" have been altered. These new results can be reinforced by agreements and possible sanctions. Although each country may seek its own self interests, it is also to their advantage to be reliable and to fulfill such treaty obligations. Although this paragraph may sound more like philosophy or theology and appear antiscientific, I would prefer to view it as a more humane, peaceful and compromising approach to the problem. And the very beginnings of mathematical theories in these latter directions are becoming available. For example, consider the private entrepreneur who attempts to develop an alternate energy technology such as oil shale or coal liquefaction. The concern is that OPEC would lower its prices and the new enterprise would fold leaving the investor holding an empty bag of money. However, if independence is worth the price and we do have the resolve, then this lowering of oil prices can be viewed as a positive gain to be redistributed among the different parties in an equitable way. These can be determined to some extent beforehand by incentives and guarantees. In chapter 10 of McDonald's book [2] on off-track betting in New York State he describes how a fifth player, New York City, entered an existing and profitable four-person game and brought with it the potential for significant additional profits, and yet threatened to ruin the whole game because of the lack of agreement on how the totality of gains should be reallocated among the five with no one being worse off than he was originally. There are many illustrations in which participants in a market type situation can obtain more by cooperative agreements than by having to depend on the less trusting equilibrium.

point approach.

It is rather obvious that the world of energy is laden with many "value" concepts, and that many less quantifiable but important parameters are being overlooked in our quantitative models. Quality of life or standard of living are not proportional to GNP nor the amount of energy consumption. Man is more than a purely political animal. Nor would I accept the assertion that my economic transactions in this constrained world truly reflect my personal desires or value system. It is stated that the standard of living in the Scandinavian countries is as high as in the U.S.A., but that they use only about half as much energy. (One could debate this last statement on grounds that the former countries import most of their energy and thus do not account for the large amount of energy expended in the producing and transportation of energy itself.) In any case, I feel that we must face up to the fact that value concepts, welfare considerations, social aspects, and in particular multiattribute utility functions must be brought into our analysis. This will likely prove to be quite difficult, since the social sciences are much richer in adjectives than the physical, mathematical and engineering sciences. On the other hand, there has recently been several advances plus a great number of reports in utility theory. These include, for example, results on when and how to decompose utility functions into lower dimensional ones by Peter Farquhar [1] -- well as interactive computer programs by Alan Sicherman [4] on how to assess one's utility function. The current indecisiveness by the U.S. Government may be interpreted in part as the difficulty in arriving at a national utility function for even one moment in time. Nevertheless, most analytical approaches, including optimization and equilibrium models, are typically weak because they fail to pull out and explicitly incorporate such value considerations.

An integral part of the world of energy is its dynamical features. Changes in bargaining, negotiations, prices, supply and demand, and cartel type behavior create a fairly continual flux. It is most important to understand the dynamics of group compromise and settlement, the ever changing behavior and value systems, the formation and cohesion of coalitions, as well as the use of incentives or inducements to bring about defections or rearrangements within alliances. One must delineate the social and economical forces which move one from the status quo point to a new position, investigate which forces need be applied to reach a desirable outcome, and whether this result is a stable one. The existing theories of bargaining, control, and differential games likely have something to offer in this direction. Some recent dynamic models in the theory of multiperson cooperative games may prove useful in such efforts; for example, work by L. J. Billera, L. S. Wang, J. H. Grotte, and R. J. Weber done at Cornell, M. Maschler and B. Peleg at the Hebrew University, as well as by others.

Some conclusions. It is quite obvious that the energy problem has a great deal of quantifiable structure, and research to date, including the presentations at this conference, demonstrate that an excellent start has been made in applying the mathematical approach to this field. It is essential that there be a great deal more communication, exchange and cooperation between different research activities, and this meeting has been helpful in this regard. The increased interest, involvement, and funding gives rise to some outstanding opportunities. However, the mathematical contribution still remains far short of its real potential, and I believe that more original modeling from scratch will prove to be an essential ingredient. The mathematization of this field is in an early stage and it remains to be seen whether it will merely plod along or explode out with new ideas. I feel that for various reasons there is



justification for some skepticism as to whether this great potential or expectations will be achieved. In my judgment, the jury is still out.

### Discussion

Snow: What about future mathematics? What types are needed?

Newlon: What breakthroughs are needed to solve the problems of energy?

Lucas: Of primary importance is the development of improved methodologies for dynamic and stochastic models. There exists already some methods of the former type which may be useful, but these are very difficult subjects. A better organized and coordinated research strategy may increase the usability of such intractable methodologies.

Much of current energy research has what I would call a "one-dimensional" character. Confrontation of the "multidimensional" nature of our problems is necessary, that is, the multiperson, multiattribute, and multiobjective aspects must be considered. Some exciting new theories could arise from such work.

There are several interesting new developments taking place in the ongoing mathematization of the social sciences, and significant additional advances could result from a more cooperative effort on the part of the mathematicians. In addition to closing this gap on our "soft" side, we must also continually search through the huge number of more recent developments in pure mathematics to find out which parts may prove useful in applications. Although a different temperament and intuition motivated the latter, there may be some pleasant surprises in terms of its usefulness.

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